Unit Vector Parallel to Tangent Line

41. Find the unit vectors that are parallel to the tangent line to the parabola $y = x^2$ at the point $(2, 4)$.

Q12.2-41 from Calculus: Early Transcendentals 7e by Stewart
Why: Need a vector in the same direction but a different length.
Steps:
1. Find the slope of the tangent.
2. Find the unit vectors parallel to the tangent.

(1) Solve for the slope of the tangent

$$\frac{dy}{dx} = 2x$$

The slope at the point (2,4) is:

$$m = \frac{dy}{dx} = 2x = 2(2) = 4$$

Extra

The equation of the line with a slope of 4 Passing through (2,4) is:

$$y = mx + b$$
$$y = 4x + b$$
$$4 = 4(2) + b$$
$$4 = 8 + b$$
$$b = -4$$

$$y = 4x - 4$$

(2) The vector must be parallel to the tangent line.

$$\vec{v} = \pm < 1,4 >$$

The unit vector is:

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{\pm < 1,4 >}{\sqrt{1^2 + 4^2}} = \pm \frac{< 1,4 >}{\sqrt{17}}$$

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