Standard Basis Vectors (Solution)

19–22 Find $a + b$, $2a + 3b$, $|a|$, and $|a - b|$.

19. $a = \langle 5, -12 \rangle$, $b = \langle -3, -6 \rangle$

20. $a = 4\,i + j$, $b = i - 2\,j$

21. $a = i + 2\,j - 3\,k$, $b = -2\,i - j + 5\,k$

22. $a = 2\,i - 4\,j + 4\,k$, $b = 2\,j - k$

(1) Calculate quantities

$\vec{a} + \vec{b} = (\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) + (-2\hat{\imath} - \hat{\jmath} + 5\hat{k}) = -\hat{\imath} + \hat{\jmath} + 2\hat{k}$
$\vec{a} + \vec{b} = -\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$2\vec{a} + 3\vec{b} = 2(\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) + 3(-2\hat{\imath} - \hat{\jmath} + 5\hat{k}) = (2\hat{\imath} + 4\hat{\jmath} - 6\hat{k}) + (-6\hat{\imath} - 3\hat{\jmath} + 15\hat{k})$
$2\vec{a} + 3\vec{b} = -4\hat{\imath} + \hat{\jmath} + 9\hat{k}$

$\vec{a} - \vec{b} = (\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) - (-2\hat{\imath} - \hat{\jmath} + 5\hat{k}) = 3\hat{\imath} + 3\hat{\jmath} - 8\hat{k}$

(2) Calculate the magnitudes

$|\vec{a}| = \sqrt{(1)^2 + (2)^2 + (-3)^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$
$|\vec{a}| = \sqrt{14}$

$|\vec{a} - \vec{b}| = \sqrt{(3)^2 + (3)^2 + (8)^2} = \sqrt{9 + 9 + 64} = \sqrt{82}$
$|\vec{a} - \vec{b}| = \sqrt{82}$

Solution to 12.2-20:

$\vec{a} + \vec{b} = (4\hat{\imath} + \hat{\jmath}) + (\hat{\imath} - 2\hat{\jmath}) = 5\hat{\imath} - \hat{\jmath}$
$\vec{a} + \vec{b} = 5\hat{\imath} - \hat{\jmath}$

$2\vec{a} + 3\vec{b} = 2(4\hat{\imath} + \hat{\jmath}) + 3(\hat{\imath} - 2\hat{\jmath}) = (8\hat{\imath} + 2\hat{\jmath}) + (3\hat{\imath} - 6\hat{\jmath})$
$2\vec{a} + 3\vec{b} = 11\hat{\imath} - 4\hat{\jmath}$

$\vec{a} - \vec{b} = (4\hat{\imath} + \hat{\jmath}) - (\hat{\imath} - 2\hat{\jmath}) = 3\hat{\imath} + 3\hat{\jmath}$

$|\vec{a}| = \sqrt{(4)^2 + (1)^2} = \sqrt{16 + 1} = \sqrt{17}$
$|\vec{a}| = \sqrt{17}$

$|\vec{a} - \vec{b}| = \sqrt{(3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18}$
$|\vec{a} - \vec{b}| = 3\sqrt{2}$