

Standard Basis Vectors (Solution)

19–22 Find $\mathbf{a} + \mathbf{b}$, $2\mathbf{a} + 3\mathbf{b}$, $|\mathbf{a}|$, and $|\mathbf{a} - \mathbf{b}|$.

19. $\mathbf{a} = \langle 5, -12 \rangle$, $\mathbf{b} = \langle -3, -6 \rangle$

20. $\mathbf{a} = 4\mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{i} - 2\mathbf{j}$

21. $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, $\mathbf{b} = -2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$

22. $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$, $\mathbf{b} = 2\mathbf{j} - \mathbf{k}$

(1) Calculate quantities

$$\vec{a} + \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + (-2\hat{i} - \hat{j} + 5\hat{k}) = -\hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{a} + \vec{b} = -\hat{i} + \hat{j} + 2\hat{k}$$

$$2\vec{a} + 3\vec{b} = 2(\hat{i} + 2\hat{j} - 3\hat{k}) + 3(-2\hat{i} - \hat{j} + 5\hat{k}) = (2\hat{i} + 4\hat{j} - 6\hat{k}) + (-6\hat{i} - 3\hat{j} + 15\hat{k})$$

$$2\vec{a} + 3\vec{b} = -4\hat{i} + \hat{j} + 9\hat{k}$$

$$\vec{a} - \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) - (-2\hat{i} - \hat{j} + 5\hat{k}) = 3\hat{i} + 3\hat{j} - 8\hat{k}$$

Solution to 12.2-20:

$$\vec{a} + \vec{b} = (4\hat{i} + \hat{j}) + (\hat{i} - 2\hat{j}) = 5\hat{i} - \hat{j}$$

$$\vec{a} + \vec{b} = 5\hat{i} - \hat{j}$$

$$2\vec{a} + 3\vec{b} = 2(4\hat{i} + \hat{j}) + 3(\hat{i} - 2\hat{j}) = (8\hat{i} + 2\hat{j}) + (3\hat{i} - 6\hat{j})$$

$$2\vec{a} + 3\vec{b} = 11\hat{i} - 4\hat{j}$$

$$\vec{a} - \vec{b} = (4\hat{i} + \hat{j}) - (\hat{i} - 2\hat{j}) = 3\hat{i} + 3\hat{j}$$

Q12.2-21 from Calculus: Early Transcendentals 7e by Stewart

Why: Want to calculate different quantities.

Steps:

1. Calculate quantities using algebraic vectors
2. Calculate the magnitude for selected quantities

(2) Calculate the magnitudes

$$|\vec{a}| = \sqrt{(1)^2 + (2)^2 + (-3)^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$$

$$|\vec{a}| = \sqrt{14}$$

$$|\vec{a} - \vec{b}| = \sqrt{(3)^2 + (3)^2 + (8)^2} = \sqrt{9 + 9 + 64} = \sqrt{82}$$

$$|\vec{a} - \vec{b}| = \sqrt{82}$$

$$|\vec{a}| = \sqrt{(4)^2 + (1)^2} = \sqrt{16 + 1} = \sqrt{17}$$

$$|\vec{a}| = \sqrt{17}$$

$$|\vec{a} - \vec{b}| = \sqrt{(3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18}$$

$$|\vec{a} - \vec{b}| = 3\sqrt{2}$$