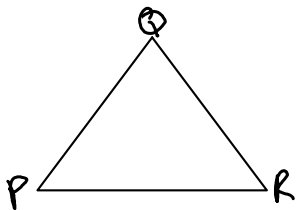# Length & Classification of Triangle

7–8 Find the lengths of the sides of the triangle PQR. Is it a right triangle? Is it an isosceles triangle?

7. $P(3, -2, -3)$, $Q(7, 0, 1)$, $R(1, 2, 1)$

8. $P(2, -1, 0)$, $Q(4, 1, 1)$, $R(4, -5, 4)$

Why: Want to calculate the length of each side of the triangle given coordinates of the vertices and classify the type of triangle.
Steps:
1. Calculate the magnitude of each side length.
2. Classify the triangle.

(1)
$$|\vec{PQ}| = \sqrt{(4-2)^2 + (1--1)^2 + (1-0)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$$
$$|\vec{QR}| = \sqrt{(4-4)^2 + (-5-1)^2 + (4-1)^2} = \sqrt{0+36+9} = \sqrt{45} = 3\sqrt{5}$$
$$|\vec{RP}| = \sqrt{(2-4)^2 + (-1--5)^2 + (0-4)} = \sqrt{4+16+16} = \sqrt{36} = 6$$

$$\boxed{|\vec{PQ}| = 3}$$
$$\boxed{|\vec{QR}| = 3\sqrt{5}}$$
$$\boxed{|\vec{RP}| = 6}$$

(2)
All three lengths are different, thus the triangle is <u>scalene</u>.
$\vec{QR}$ is the largest side lenth (hypotenues). Check if $a^2 + b^2 = c^2$

$$|\vec{PQ}|^2 + \left|\vec{RP}\right|^2 = \left|\vec{QR}\right|^2$$
$$3^2 + 6^2 = \left(3\sqrt{5}\right)^2$$
$$45 = 45$$

Thus, ΔPQR is a right triangle.

Solution to 12.1-7:

$$|\vec{PQ}| = \sqrt{(7-3)^2 + (0--2)^2 + (1--3)^2} = \sqrt{16+4+16} = \sqrt{36} = 6$$
$$|\vec{QR}| = \sqrt{(1-7)^2 + (2-0)^2 + (1-1)^2} = \sqrt{36+4+0} = \sqrt{40} = 2\sqrt{10}$$
$$|\vec{RP}| = \sqrt{(1-3)^2 + (2--2)^2 + (1--3)} = \sqrt{4+16+16} = \sqrt{36} = 6$$

Since $\left|\vec{PQ}\right| = \left|\vec{RP}\right| \neq \left|\vec{QR}\right|$, and $|\vec{PQ}|^2 + \left|\vec{RP}\right|^2 \neq \left|\vec{QR}\right|^2$,

ΔPQR is an isosceles triange.

Check if it's a right triangle:

$$|\vec{PQ}|^2 + \left|\vec{RP}\right|^2 = \left|\vec{QR}\right|^2$$
$$6^2 + 6^2 = \left(2\sqrt{10}\right)^2$$
$$36 \neq 40$$

$$\boxed{|\vec{PQ}| = 6}$$
$$\boxed{|\vec{QR}| = 2\sqrt{10}}$$
$$\boxed{|\vec{RP}| = 6}$$