

Angle Between Lines

29–30 Find the acute angle between the lines.

29. $2x - y = 3, \quad 3x + y = 7$

30. $x + 2y = 7, \quad 5x - y = 2$

(1) Slope of each line:

$$\begin{aligned} 2x - y &= 3 \\ y &= 2x - 3 \end{aligned}$$

$$m_1 = 2$$

$$\begin{aligned} 3x + y &= 7 \\ y &= -3x + 7 \end{aligned}$$

$$m_2 = -3$$

(2) Use the dot product formula

$$\begin{aligned} \vec{a} \cdot \vec{b} &= |\vec{a}| |\vec{b}| \cos \theta = a_x b_x + a_y b_y \\ \Rightarrow \cos \theta &= \frac{a_x b_x + a_y b_y}{|\vec{a}| |\vec{b}|} \end{aligned}$$

$$\vec{a} = \langle 1, 2 \rangle, \vec{b} = \langle 1, -3 \rangle$$

$$\begin{aligned} \cos \theta &= \frac{(1)(1) + (2)(-3)}{\sqrt{(1)^2 + (2)^2} \sqrt{(1)^2 + (-3)^2}} \\ \cos \theta &= \frac{-5}{\sqrt{5}\sqrt{10}} = -\frac{5}{\sqrt{50}} = -\frac{5}{\sqrt{25}\sqrt{2}} = -\frac{1}{\sqrt{2}} \\ \theta &= \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = 135^\circ \\ 180^\circ - 135^\circ &= 45^\circ \end{aligned}$$

Q12.3-29 from Calculus: Early Transcendentals 7e by Stewart

Why: Find the dot product between two vectors.

Steps:

1. Find the slope of each line.
2. Use the dot product formula to calculate the angle between the lines.